NOTATION

τ, time; t, temperature; ϑ, temperature difference; b, heating rate; q, thermal flux; Q, heat absorbed; λ, thermal conductivity; a, thermal diffusivity; c, specific heat; ρ, density; ε_a, thermal activity; h, specimen thickness; r_0, contact spot radius.

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CERTAIN PROBLEMS OF FLUID FILTRATION IN ELASTIC CRACKED-POROUS RESEVOIRS

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UDC 532.546

This article examines a procedure for realizing an integral method for filtration processes in an elastically compressible cracked-porous bed.

1. Filtration in a cracked-porous medium (a medium with two systems of channels differing significantly in permeability) is usually modeled on the basis of the representation of two interpenetrating continua which exchange mass. The model in [1] is widely used in filtration theory and practice. The rsults of the experiments conducted in [2] show, however, that permeability is more heavily dependent on the stress state of the system and fluid pressure in the cracks than is indicated in the representations in [1]. A more complete accounting of the effect of the stress state of the medium on filtration was made in [3]. Here, for the model in [3], we examine certain problems dealing with nonsteady filtration of a fluid toward a well.

2. The below equations [3] describe filtration in the elastic material of an isotropic structure in a state of cubic compression with the stress σ (as well as in the case of two-dimensional filtration in a material in which all cracks are oriented in the plane of motion)

$$a\frac{\partial\psi_1}{\partial t}=\frac{1}{4}\nabla^2(\psi_1+1)^4+\psi_2-\psi_1,\quad \frac{\partial\psi_2}{\partial t}=\varepsilon\nabla^2\psi_2-\psi_2+\psi_1, \tag{1}$$

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where

$$\psi_i = -(p^\circ - p_i)(p^\circ - \sigma)^{-1},\ r = x(\varkappa_1\tau)^{-1/2},\ t = \omega\tau^{-1},$$
$$a = m_1^\circ [m_2^\circ (p^\circ - \sigma)(\beta_\rho + \beta_m)]^{-1},\quad \varepsilon = k_2^\circ / k_1^\circ = \varkappa_2/\varkappa_1 \ll 1,$$
$$\varkappa_2 = k_2^\circ [m_2^\circ \mu^\circ (\beta_\rho + \beta_m)]^{-1}. \tag{2}$$

System (1) is valid only at $p_1 > \sigma$ ($-1 < \gamma \leq 0$), when the cracks are open. At $p_1 \leq \sigma$, the cracks are closed, and filtration of the fluid in this zone occurs only through blocks (equation of the elastic regime). Contact conditions [4] must be satisfied at the unknown boundary between the zones (where $p_1 = \sigma$).

Below we examine the case of cylindrical symmetry. However, the main conclusions reached are also valid for other symmetry variants. The results are represented in Figs. 1-3 (all of the parameters in the figure are dimensionless).

3. There are no similarity solutions for system (1). An integral method (see [5-8]) can be used to construct approximate solutions.

The steady-state solution of Eqs. (1) at $\varepsilon \to 0$ has the form

$$(\psi_i + 1)^4 = \alpha \ln r + \beta. \tag{3}$$

In accordance with the integral method [7], solution (1), with allowance for [3], is sought in the form (the permeability of the blocks is not considered)

$$(\psi_i + 1)^4 = \alpha_i \ln (r/l_i(t)) + \beta_i + \gamma_i r/l_i(t) + \ldots, \tag{4}$$

where we have introduced two boundaries $\ell_i = \ell_i(t)$ between perturbation zones which propagate along cracks and blocks [9]. For a finite deposit of radius R, the process is broken down into three phases. The moment of completion of the first phase is determined by the equation

$$l_1(t_1) = R, \tag{5}$$

while the end of the second phase is found from the equation

$$l_2(t_2) = R. \tag{6}$$

In the first phase ($t < t_1$), the pressures have the form (4) (this is also the solution for an infinite region), while in the second phase ($t_1 \leq t < t_2$), they have the same form (4) but with the substitution $\ell_1 = R$. In the third phase ($t \geq t_2$), they have the form of (4) with the substitution $\ell_1 = \ell_2 = R$.

The coefficients α, β, and γ and the laws of motion of the boundaries $\ell_i = \ell_i(t)$ in Eq. (4) are determined from the boundary conditions and integral relations corresponding to (1):

$$a \frac{d}{dt} \int_0^{\delta_1(t)} r\psi_1 dr = \lambda + \int_0^{\delta_2(t)} r\psi_2 dr - \int_0^{\delta_1(t)} r\psi_1 dr,$$
$$\frac{d}{dt} \int_0^{\delta_2(t)} r\psi_2 dr = -\int_0^{\delta_2(t)} r\psi_2 dr + \int_0^{\delta_1(t)} r\psi_1 dr. \tag{7}$$

In (7), for the first phase $\delta_i(t) = \ell_i(t)$ ($i = 1, 2$), $\lambda = -q$ (startup of a well with a constant yield q); in the second phase $\delta_1(t) = R$, $\delta_2(t) = \ell_2(t)$, and $\lambda = -q$ for a closed bed (depletion regime), while for an open bed (constant pressure maintained at the contour of the bed)

$$\lambda = -q + \frac{R}{4} [\partial (\psi_1 + 1)^4 / \partial r]_{r=R}.$$

In the third phase $\delta_i(t) = R$ ($i = 1, 2$), and the parameter λ is determined in the same manner as in the second phase.

The solution of Eqs. (7) must satisfy the following initial conditions: in the first phase

$$l_i(0) = 0, \tag{8}$$

in the second phase*

$$\psi_{i1}(t_1) = \psi_{i2}(t_1), \tag{9}$$

in the third phase

$$\psi_{i2}(t_2) = \psi_{i3}(t_2). \tag{10}$$

In determining the pressures ψ_i, we will henceforth restrict ourselves to the terms written out in Eq. (4). In this case, we have the following in the case of the problem of the startup of a well with a constant yield q for any region (finite or infinite), where the permeability of the blocks is ignored

$$\alpha_{ij} = -\gamma_{i1} = -\gamma_{22} = 4q, \quad \beta_{i1} = \beta_{22} = 1 + 4q. \tag{11}$$

For a finite bed $\beta_{12} = \beta_{12}(t)$, $\beta_{i3} = \beta_{i3}(t)$. Here, in the case of a closed bed

$$\gamma_{12} = \gamma_{i3} = -4q, \tag{12}$$

while for an open bed

$$\gamma_{12} = 1 - \beta_{12}(t), \quad \gamma_{i3} = 1 - \beta_{i3}(t). \tag{13}$$

With allowance for Eq. (4), integral relations (7) are represented in a single form for all phases of the process†

$$a\frac{dy_{1j}}{dt} = -y_{1j} + y_{2j} + \frac{\gamma_{1j}}{4}, \quad \frac{dy_{2j}}{dt} = y_{1j} - y_{2j}, \tag{14}$$

where

$$y_{ij} = (I_{ij} - 0{,}5)\,\delta_i^2, \quad I_{ij} = \int_0^1 u\,(4q\ln u + \beta_{ij} + \gamma_{ij}u)^{1/4}du, \tag{15}$$

while the coefficients β and γ take the values (11-13), depending on the type of bed.

For infinite and finite closed beds, the general solution of system (14) has the form:

$$\begin{aligned} y_{1j}(t) &= C_{1j} + C_{2j}\exp(st) + \gamma_{1j}(t + 1/b)/(4b), \\ y_{2j}(t) &= C_{1j} - C_{2j}a\exp(st) + \gamma_{1j}(t - a/b)/(4b), \end{aligned} \tag{16}$$

where s = −b/a, b = 1 + a.

For a finite open bed, integration of system (14) poses serious difficulties for the second and third phases because the terms γ_{12} and γ_{13} depend on the unknown functions β_{12} and β_{13}, respectively. In this case, we can use the method of successive approximations, determining the discontinuities γ_{12} and γ_{13} in (14) from the previous step (we take the solution for the first phase as the initial approximation). The flow of fluid into the bed necessary to maintian constant pressure at r = R is found from system (14).

4. For the first time (any filtration region) in (16)

$$C_{11} = 0, \quad C_{21} = q/b^2, \tag{17}$$

with the increase in the size of the perturbation zones being determined from the expressions:

$$\begin{aligned} [I_1(q) - 0{,}5]\,l_{11}^2 &= -q\,[t + (1 - \exp(st))/b]/b, \\ [I_1(q) - 0{,}5]\,l_{21}^2 &= -q\,[t - a(1 - \exp(st))/b]/b, \end{aligned} \tag{18}$$

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*We will henceforth use notation of the form f_{ij}, where the subscript i = 1, 2 determines the medium (cracks, blocks) and the subscript j = 1, 2, 3 determines the phase of the process.
†In the case of plane symmetry (tunnel), the integral relations reduce to the form (14). The same result can obviously be expected in the case of spherical symmetry as well.

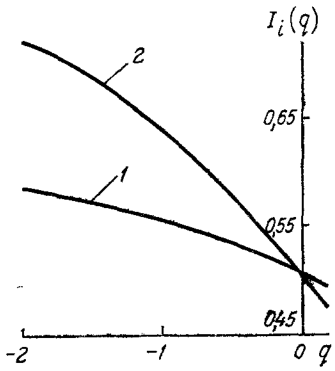

Fig. 1. The relations $I_1(q)$ (1) and $I_2(q)$ (2).

where

$$I_1(q) = I_{11}(q) = I_{21}(q) = \int_0^1 u\,[1 + 4q(1 + \ln u - u)]^{1/4}\,du. \tag{19}$$

From estimates of the parameters which determine the dimensionless yield

$$q = \mu Q\,[2\pi h k_1^\circ (p^\circ - \sigma)]^{-1},$$

it follows that we nearly have $0 < q \leq 0.1$. The function $K(q) = q/(0.5-I_1(q))$ is bounded and monotonically decreasing; for the actual values being examined for q: $-2 \leq q \leq 0.1$ ($q < 0$ – injection, $q > 0$ – extraction) $9.475 \leq K(q) \leq 3.1$ (see Fig. 1). Thus, with an increase in fluid extraction, the motion of the boundaries $\ell_{i1} = \ell_{i1}(t)$ is slowed. This occurs because an increase in yield is accompanied by an increase in the rate of reduction in bed pressure, which in turn results in greater compression of the cracks that are present.

At small values of time $l_{11}^2 \approx l_{21}^2 \approx qt/[b(0{,}5 - I_1(q))]$. Then the boundary ℓ_{11} overtakes the boundary ℓ_{21}. At sufficiently large values of time (infinite bed), a constant distance is established between the boundaries of the perturbation zones

$$l_{11}^2 - l_{21}^2 \approx q\,[b\,(0{,}5 - I_1(q))]^{-1} \tag{20}$$

Accordingly, with small values of time, the pressure distributions in the cracks and block (and the rate of pressure reduction) are fairly close. The difference in the rates of pressure reduction increases with an increase in time.

Thus, the first phase includes flow of fluid from the blocks into the cracks. This flow increases from zero to a maximum value corresponding to the constant difference (2).

In the second and third phases for a closed bed, the values in (17) and the constants C_{ij} in (16) remain as before.

In the zone $\ell_{21} \leq r \leq \ell_{11}$, the initial zero pressure is maintained in the blocks, while pressure decreases in the cracks. From a physical viewpoint, this is explained by the difference in the rates of propagation of perturbations via the cracks and blocks. The same conclusion follows from (21) (see below – a similar conclusion canbe reached on the basis of (4)); for fixed r, it follows from the condition $\ell_2(t) < \ell_1(t)$ that $\psi_2 > \psi_1$.

5. With the selection of more approximate profiles for the first phase

$$(\psi_i + 1)^4 = 1 + 4q \ln(r/l_i(t)) \tag{21}$$

the integral $I_1(q)$ in Eqs. (18) and (19) is replaced by $I_2(q)$:

$$I_2(q) = \int_0^1 u\,(1 + 4q \ln u)^{1/4}\,du. \tag{22}$$

In this case, we find the boundary ℓ_{22} for the second phase from Eqs. (14) in the case of a closed bed (the form of (21) remains the same except for the substitution $\ell_{11} = R$):

$$l_{22}^2(t) = R^2 - q\varphi(t,\ t_1)[0{,}5 - I_2(q)]^{-1},$$

and we find the flow into the bed at r = R needed to maintain a constant contour pressure:

$$\frac{R}{4}[\partial(\psi_1+1)^4/\partial r]_{r=R}=q[1-\varphi(t,\ t_1)],$$

where

$$\varphi(t,\ t_1)=[1-\exp(st_1)]\exp(t_1-t)/b.$$

The moment = t_1 of completion of the first phase is determined by Eqs. (8) and (18). Flow into the bed increases with time and approaches q, which is a consequence of ignoring the permeability of the blocks.

The form (21) of the function ψ_2 remains the same for the second phase in the case of a closed bed, while the function ψ_1 must be constructed in the form (4):

$$(\psi_1+1)^4=4q[\ln(r/R)-r/R]+\beta_{12}(t), \tag{23}$$

since use of the initial profile (21) leads to a contradiction in the solution of Eqs. (14). In Eqs. (16), for the second phase γ_{12} = −4q,

$$y_{12}(t)=R^2\Big[\int\limits_0^1 u(4q\ln u-4qu+\beta_{12}(t))^{1/4}du-0{,}5\Big],$$

$$y_{22}(t)=[I_2(q)-0{,}5]\,l_{22}^2(t). \tag{24}$$

Since we chose basically different profiles of ψ_1 for the first and second phases, the first condition of (9) can be satisfied only at r = R, which is physically valid. The constants C_{i2} for the second phase have the values:

$$C_{12}=(ad_2+c_2)/b,\quad C_{22}=(d_2-c_2)\exp(-st_1)/b, \tag{25}$$

$$c_2=-qa\exp(st_1)/b^2,\quad d_2=R^2(I_1(q)-0{,}5)+q/b^2+qt_1/b. \tag{26}$$

For the third phase, the pressures ψ_i have the form (23) and the unknown functions $\beta_{i3}(t)$ are determined by Eqs. (16), where γ_{13} = −4q

$$y_{i3}(t)=R^2\Big[\int\limits_0^1 u(4q\ln u-4qu+\beta_{i3}(t))^{1/4}du-0{,}5\Big], \tag{27}$$

while the constants C_{i3} have the form (25), with the replacement of c_2, d_2 by c_3, d_3, respectively:

$$c_3=-qa/b^2+qt_2/b+R^2[I_1(q)-0{,}5],\quad d_3=C_{12}+C_{22}\exp(st_2). \tag{28}$$

The unknown functions $\beta_{ij}(t)$ in integrals (15) (also see (24), (27)) can be determined numerically by using the results of tabulation of the integrals (19), (22) (Fig. 1). Integrals of this type were introduced in [10] to study gas filtration. It should be noted that although $I_i(q) \approx 0.5$ (for q > 0), the order of the terms $(I_i(q) - 0.5)\ell_i^2$, $(I_i(q) - 0.5)R^2$ in Eqs. (24-28) can have any value.

6. The time of completion of the phases t_1, t_2 depends on the dimensionless complexes R, a, q, and ε and was calculated numerically from Eqs. (5) and (6) for the general case.

It follows from the numerical calculations (Fig. 2) and a qualitative analysis that the values of t_1 and t_2 increase with an increase in the parameters R, a, and q (their ranges are determined on the basis of estimates of the initial characterisics of the system). The complexes R and a have the greatest effect: an increase in R by one order of magnitude increases t_i by two orders for most of the values of a and q examined; an increase in a by four orders increases t_i by four orders for R = 10^{-1}, 1 and by two orders for R = 10, 10^2, 10^3. The dependence of t_i on the yield is weaker: an increase in q from 0.01 to 0.1 increases t_i by an average of 30%. The parameter ε has a negligible effect on the time of completion of the phases. Given sufficiently small values of the complexes R and a, a perturbation is propagated considerably more rapidly by the cracks than by the blocks (the

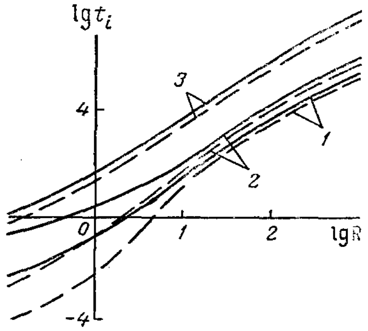

Fig. 2. Dependence of the time of completion of the phases of the process (t_1 – dashed curves, t_2 – solid curves) on the parameter R: a = 10^{-2} (1); 1 (2); 10^2 (3).

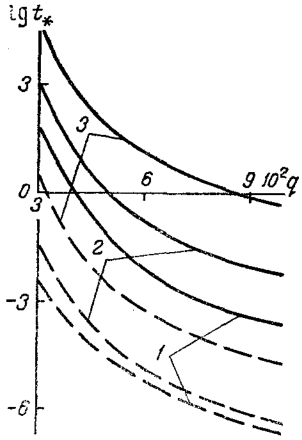

Fig. 3. Dependence of the time of closure of cracks on the well ($r_0 = 10^{-4}$ – dashed curves; $r_0 = 10^{-2}$ – solid curves) on the yield: a = 10^{-2} (1); 1 (2); 10^2 (3).

times t_1 and t_2 may differ by two orders of magnitude). At large values of R and a, the times t_i are of the same order.

The dimensional time of completion of the phases $\omega_i \ll \tau$ (or $\sim\tau$) only for sufficiently small values of the parameters R and a.

7. In regard to the depletion problem, it is of practical interest to evaluate the time $t = t_*$, when the fluid pressure on the well drops to the critical value σ ($\psi_1 = -1$). The cracks close at this pressure. This time can be roughly determined by using Eqs. (21) or (4) for ψ_1. For example, for profile (21), the value $t = t_*$ is the root of the equation

$$l_{11}(t_*) = \tau_0 \exp(1/4q), \tag{29}$$

where r_0 is the dimensionless radius of the well and ℓ_{11} is determined from the first formula in (18). Taking the radius of the well equal to 0.1 m and taking into account the ranges of the parameters κ_1 and τ over which the dimensionless length is determined (see (2)), we find $r_0 \sim 10^{-4}$–10^2 (it should be noted that $r_0 = 10^{-4}$ corresponds to the dimension of the bed R = 10^{-1} – 1, $r_0 = 10^{-3}$ – R = 1 – 10, $r_0 = 10^{-2}$ – R = 10 – 10^2).

If we compare Eq. (29) with Eq. (5) for the moment $t = t_1$ corresponding to the end of the first phase, we find that $t_* \le t_1$ at values of yield satisfying the condition

$$\exp(1/4q) \leqslant R/r_0. \tag{30}$$

For small values of yield, the cracks on the well close in the second or third phases, and there is a corresponding change in Eq. (29).

Figure 3 shows results of calculations of the moment $t = t_*$ from Eq. (29). The moment of crack closure is heavily dependent on the yield: the time $t = t_*$ decreases by several orders of magnitude with an increase in the yield from 0.03 to 0.1. The part of Fig. 3 located below the yield axis corresponds to the dimensional time $\omega_* \le \tau$.

A qualitative representation of the motion of the front of crack closure $r = r_*(t)$ can be obtained from Eq. (21):

$$r_*(t)\exp(-1/4q) l_{11}(t), \quad t > t_*. \tag{31}$$

It should be emphasized that Eqs. (29-31) give only an approximate picture of the process, since the permeability of thr cracks becomes comparable to the permeability of the blocks after the cracks close, and the permeability of the blocks was not established here. The problem of depletion with allowance for the crack closure front, to be examined later, is an analog of the Stefan problem.

NOTATION

p, x, ω and ψ, r, t, dimensional and dimensionless pressures, coordinates, and times, respectively; Q, q, dimensional and dimensionless yields; k, m, κ, permeability, porosity, and piezoelectric conductivity; h, capacity of the bed; τ, lag time; β_ρ, β_m, elastic moduli; μ, viscosity of the fluid; t_i, time of completion of the phases; t_*, time of crack closure; ∇^2, Laplace operator; the symbol f° denotes characteristics of the system in the presence of initial pressure p°. The indices 1 and 2 pertain to the cracks and blocks, respectively.

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